Hale School

Mathematics Specialist

Test 3 --- Term 2 2016

Vectors in 3D

Name: _____

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Instructions:

- **CAS calculators are allowed**
- **External notes are not allowed**
- **Duration of test: 50 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

Question 1 (7 marks: 1, 2, 4)

A sphere has its centre at C(1, 1, 0) and radius 3.

(a) State the Cartesian equation of the sphere.

Consider a diameter with end points at P(3, 3, 1) and Q(a, b, c) on the sphere.

(b) Determine the values of a, b and c.

Let X(x, y, z) be any point (except P and Q) on the sphere.

(c) Prove that **PX** is perpendicular to **QX**.

Question 2 (3 marks: 1, 2)

The Cartesian equation of a plane π is given by $x - 2y + z = 3$ and the vector equation of a line ℓ is given by $\vec{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$.

(a) State a normal vector to π.

(b) State the vector equation of the plane Λ which contains the line ℓ and parallel to π.

Question 3 (6 marks: 2, 4)

An object, A, with initial position vector $\mathbf{r}_A(0) = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ metres is moving with velocity $\mathbf{v}_A = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ m/s.

A second object, B, with initial position vector $\mathbf{r}_B(0) = 5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ metres is moving with velocity $\mathbf{v}_B = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ m/s.

(a) Find the positions of A and B at time t.

(b) If $|\mathbf{v}_B| = \sqrt{14}$ m/s and A and B collide, find the time(s) of collision.

Question 4 (7 marks: 4, 2, 1)

Given the equations of two planes: $\pi_1 : x - y + z = 1$ and $\pi_2 : x - z = 4$.

(a) Find the vector equation of the line which π_1 and π_2 intersect.

A third plane is given by $\pi_3 : 4x - 3y + 2z = d$ where d is an unknown.

(b) (i) Determine the value of d if the three equations $\begin{cases} x - y + z = 1 \\ x - z = 4 \\ 4x - 3y + 2z = d \end{cases}$ have many solutions.

(ii) Given the solutions in (b) (i), provide a geometric interpretation of the three planes in (b) (i).

Question 5 (4 marks: 3, 1)

Given three vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 4 \\ -7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 0 \\ -9 \\ h \end{pmatrix}$ where h is an unknown.

(a) Given that $\mathbf{b} \times \mathbf{c} = 18\,\mathbf{i} - 36\,\mathbf{j} - 18\,\mathbf{k}$, determine the value of h.

(b) Evaluate $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$.

(c) Give a geometric interpretation regarding the three vectors of your answer in part (b).

Question 6 (7 marks: 2, 2, 3)

A particle moves along a path described by the vector function $\mathbf{r}(t) = 2\sin(\frac{t}{2})\mathbf{i} + 3\cos(\frac{t}{2})\mathbf{j}$ for $0 \leq t \leq 2\pi$.

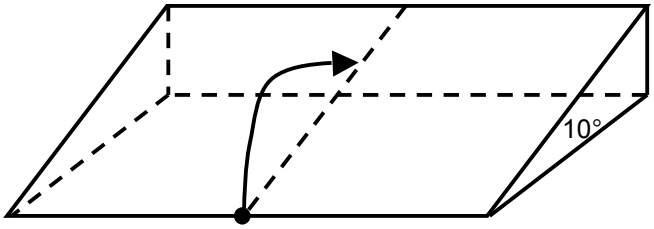
(a) Determine the Cartesian equation of the path.

(b) Determine the velocity function.

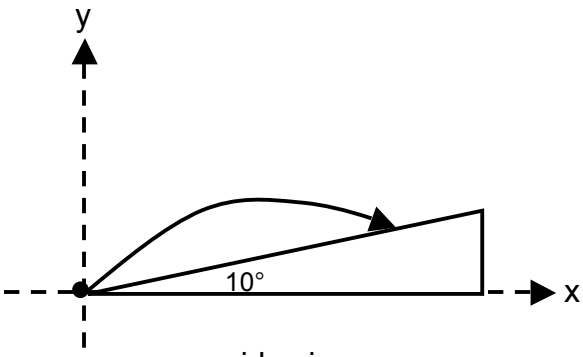
(c) Determine the maximum speed.

Question 7 (7 marks: 3, 2, 2)

A particle, at the bottom of an inclined plane, is projected up the plane along a line of the greatest slope as shown below.



3D view



side view

The initial velocity of the particle is 40 m/s making an angle 20° with the **plane**.

The particle experiences an acceleration of 10 m/s^2 downwards throughout its motion. Ignore air resistance and use vector calculus in answering the following questions.

(a) Determine the velocity vector of the particle at time t.

(b) Determine the position vector of the particle at time t.

(c) Determine the time when the particle hits the plane.